**Estimating missing data with** **Machine Learning when Know correlation and Small Sample Sizes**

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**Abstract.** This paper presents a comparative analysis of K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF) for estimating loss values under varying conditions of missing data (5%, 10%, 15%) and correlation coefficients (ρ). The study aims to determine which method performs best under different scenarios of data sparsity and correlation. Our methodology involves calculating the average absolute error (AAE) for each method across different rates of missing data and ρ values. The results indicate that SVR achieves the lowest AAE at lower missing data rates and lower ρ values, whereas RF excels as the rate of missing data and ρ increase. Specifically, RF demonstrates superior performance with the lowest AAEs at higher missing data rates and higher ρ values, making it the most reliable method overall. The discussion highlights the robustness of RF in handling incomplete and correlated datasets, and its consistent performance compared to other methods. The study concludes by suggesting future research directions, including the development of hybrid models that combine the strengths of SVR and RF, and the exploration of various imputation techniques to enhance model performance. These findings are significant for improving loss estimation and decision-making in fields such as finance, healthcare, and engineering.

**Keywords.** Data Imputation, Decision Trees, K-Nearest Neighbors, Machine Learning, Random Forests, Support Vector Regression

# Introduction

In the domain of data analysis and machine learning, managing missing data is a significant challenge that can substantially impact the reliability and quality of conclusions. Missing data can arise from various sources such as errors in data collection, transmission issues, or non-responses in surveys. Addressing missing data appropriately is crucial because it can lead to biased estimates, diminished statistical power, and potentially incorrect conclusions [1]. Missing data mechanisms are generally classified into three categories: Missing Completely at Random (MCAR), Missing at Random (MAR), and Missing Not at Random (MNAR) [2]. MCAR occurs when the probability of missing data on a variable is independent of any observed or unobserved data. MAR occurs when the probability of missing data is related to observed data but not the missing data itself. MNAR occurs when the missingness is related to the value of the variable itself.

The effects of correlated missing data are multifaceted. Firstly, it can introduce bias in parameter estimates. If missing data are correlated with other variables, ignoring this missingness can result in biased parameter estimates. For example, if higher values are more likely to be missing, the mean estimate of the dataset will be biased downward. Secondly, missing data reduce the effective sample size, increasing the variance of estimates and reducing the efficiency of statistical tests, potentially leading to incorrect conclusions. Thirdly, correlation patterns among variables with missing data can complicate the modeling process, particularly if the data are MNAR, necessitating sophisticated models to properly account for the mechanisms behind the missingness [3].

Imputation is a common method for handling missing data, involving substituting missing values with plausible estimates based on other available data. Simple imputation techniques include using the mean, median, or mode of the observed values. More sophisticated methods involve model-based approaches like multiple imputation, where missing values are filled in several times to create different complete datasets [4]. For instance, [5] categorize various imputation methods used in data mining, providing insights into their applications and effectiveness depending on the type of missing data and the specific requirements of the analysis [5].

Recent advancements in machine learning offer new approaches for handling missing data. Techniques such as k-nearest neighbors (k-NN), decision trees, and deep learning algorithms can inherently manage missing values or be adapted to accommodate incomplete datasets [6]. Research on k-nearest neighbors (k-NN) for missing data imputation highlights several significant advancements and comparative studies. [7] introduced an interval value imputation method that enhances the efficiency of k-NN by using the Euclidean distance to find the k closest complete data points for imputing missing values. This method proved to be more effective than traditional imputation techniques like zero-value and median imputation [7]. [8] explored the use of an optimized fuzzy clustering-based k-NN imputation (FC-KNNI) for handling mixed missing data in software development effort estimation (SDEE). Their findings indicated that FC-KNNI significantly outperformed classical k-NN imputation, particularly in dealing with categorical data, enhancing the accuracy of SDEE techniques [8]. Another study proposed new imputation methods, such as LMKNN\_Imputation and Weighted KNN\_Imputation, which leverage local mean vectors and weighted averages to impute missing data. These methods demonstrated high accuracy in predicting missing values and improved classification performance. Lastly, a new clustering method integrating fuzzy C-means with k-NN imputation was introduced. This approach treats missing values as optimization variables within the objective function, leading to superior clustering results for incomplete datasets.

Decision tree regression is another versatile and powerful machine learning technique that offers both clarity and accuracy in predicting continuous outcomes. While it has some limitations, such as susceptibility to overfitting, these can often be managed through techniques like pruning or by leveraging ensemble methods. With its intuitive model structure, decision tree regression remains a popular choice among data scientists for solving complex regression problems [9].

Random Forest Regression is a powerful, versatile, and widely used machine learning technique. Its ability to handle large datasets with numerous predictor variables, combined with its robustness to overfitting and high accuracy, makes it a popular choice for regression tasks across many industries. While it may not offer the same level of simplicity and interpretability as single decision trees, its advantages often outweigh these drawbacks [10].

Support Vector Regression stands out as a robust method for regression analysis, capable of handling both linear and non-linear relationships with high accuracy. Its applicability across diverse domains underscores its versatility and power. Despite challenges in parameter tuning and computational demands, SVR remains a valuable tool for predictive analytics where precision is critical [11].

This research study focuses on innovative techniques and methodologies for handling missing data in the context of machine learning and data analysis. It advances the state-of-the-art by providing a comprehensive analysis of the accuracy of K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF) in estimating loss values under varying conditions of missing data and correlation coefficients (ρ).

# Methods

## Research methods

This research compares the accuracy of missing data imputation using four methods as k-NN, decision tree regression, Random Forest Regression, and Support Vector Regression for missing data. The study employs Monte Carlo Simulation using R software with the following steps:

Step 1: Define the ρ levels between X and Y as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

Step 2: Generate a population of 5,000 units comprising the independent variable (X) and the dependent variable (Y), both normally distributed with a mean of 10 and a variance of 1, and the specified correlation levels (ρ) between X and Y from step 1.

Step 3: Perform simple random sampling from the population with sample sizes of 30, 50, 70, and 90.

Step 4: Introduce missing values in the dependent variable Y at rates of 5%, 10%, and 15%.

Step 5: Impute the missing values using k-NN, decision tree regression, Random Forest Regression, and Support Vector Regression.

Step 6: Calculate Average Absolute Error (AAE) to assess the imputation accuracy.

$$AAE=\frac{\sum\_{i=1}^{n}\left|\hat{y}\_{i}-y\_{i}\right|}{n}$$

Step 7: Repeat the process 10,000 times from steps (3) to (6), then calculate AAE.

We define that $y\_{1},y\_{2},y\_{3},...,y\_{r}$ represents the complete data values, $y\_{r+1},y\_{r+2},y\_{r+3},...,y\_{n}$​ represents the missing data values, and all $x\_{i}$ ; $i=1,2,3,...,n$ are complete. The k-NN, decision tree regression, Random Forest Regression, and Support Vector Regression for missing data are as follows.

## k-Nearest Neighbors

The k-Nearest Neighbors (k-NN) algorithm is a non-parametric method used to impute missing data by leveraging the similarity between data points. Steps of K-NN are as follows [12].

Step 1: Select the number of nearest neighbors k to use for the imputation. This parameter k can be optimized using cross-validation.

Step 2: Calculate Distances. For each observation i with a missing $y\_{r+1},y\_{r+2},y\_{r+3},...,y\_{n}$ values, compute the distance between this observation and all other observations with complete $y\_{1},y\_{2},y\_{3},...,y\_{r}$ values. The Euclidean distance is commonly used as

$$d(i,j)=\sqrt{\sum\_{l=1}^{n}(x\_{il}-x\_{jl})^{2}}$$

where $x\_{il}$ and $x\_{jl}$ are for observations i and j, respectively.

Step 3: Select Nearest Neighbors. Identify the k nearest neighbors (those with the smallest distances) for each observation with a missing value.

Step 4: Impute the Missing Value by using the mean as

$$\hat{y}=\frac{1}{k}\sum\_{i=1}^{k}y\_{i}$$

This structured approach ensures that k-NN effectively utilizes the nearest data points to input the missing value.

## Support Vector Regression

Support Vector Regression (SVR) is a powerful and flexible method for imputing missing data, particularly when dealing with complex datasets. Steps of the SVR Algorithm are as follows [13].

Step 1: Support Vector Regression aims to find a function f(x) that has at most ϵ epsilonϵ deviation from the actual target values y for all the training data, and at the same time is as flat as possible.

The function f(x) is defined as:

$$f\left(x\right)=\left〈w,x\_{i}\right〉+b$$

where $\left〈w,x\_{i}\right〉$ is the dot product of w and x, and b is a bias term.

Step 2: The SVR optimization problem can be formulated as:

$$min\_{W,b,ξ,ξ^{\*}}\frac{1}{2}\left‖w\right‖^{2}+C\sum\_{i=1}^{n}ξ\_{i}+ξ\_{i}^{\*}$$

subject to

$$\left\{\begin{matrix}y\_{i}-\left〈w,x\_{i}\right〉+b\leq ϵ+ξ\_{i}\\\left〈w,x\_{i}\right〉+b-y\_{i}\leq ϵ+ξ\_{i}^{\*}\\ξ\_{i},ξ\_{i}^{\*}\geq 0\end{matrix}\right.$$

where $ξ\_{i}$ and $ξ\_{i}^{\*}$​ are slack variables that allow for some errors, and C is a regularization parameter that determines the trade-off between the flatness of f(x) and the amount tolerated by the deviations larger than ϵ.

Step 3: SVR can be extended using kernel functions $K(x\_{i},x\_{i})$. The kernel function maps the input space into a higher-dimensional feature space where linear regression is performed as

$$K\left(x\_{i},x\_{i}\right)=\left〈x\_{i},x\_{i}\right〉$$

Step 4: Impute Missing Values. Train the SVR model on the dataset with complete y values. Then, use the trained SVR model to predict (impute) the missing y values.

## Decision Trees

Decision Trees (DT) are a versatile and intuitive method for imputing missing data. Steps of DT Algorithm are as follows [9].

Step 1: Train the Decision Tree Model. Use the complete data to train the Decision Tree model.

Step 2: Splitting Criterion:

The tree splits the data at each node based on the attribute that minimizes the impurity or error. For regression tasks, the impurity measure is often the Mean Squared Error (MSE).

$$MSE=\frac{\sum\_{i=1}^{n}\left(\hat{y}\_{i}-y\_{i}\right)^{2}}{n}$$

where $y\_{i}$ are the actual values, and ​ $\hat{y}\_{i} $are the predicted values.

Step 3: Recursive Partitioning. The Decision Tree recursively partitions the data into subsets based on the attribute values, creating nodes and leaves. The split at each node is determined by the attribute that results in the lowest MSE.

Splitting Rule:

$$split\rightarrow \left\{\begin{array}{c}Left subtree if x\_{i}\leq Threshold\\Right subtree if x\_{i}>Threshold\end{array}\right.$$

where:

xi is the value of the attribute used for the split.

Threshold is the value that minimizes the MSE for the split.

Step 4: Apply the trained Decision Tree model to predict the missing values in yi using the corresponding xi values.

For each observation iii with a missing value in y:

 Traverse the tree from the root to a leaf node using the xi values.

 The predicted value is the mean value of yi in the corresponding leaf node.

## Random Forest

Random Forest is an effective method for imputing missing data by combining predictions from multiple decision trees. Steps of RF are as follows.

Step 1: Generate B bootstrap samples from the original dataset. Each sample is created by randomly sampling n observations with replacement from the original data of size n.

Mathematically, for each bootstrap sample Sb​:

$$S\_{b}=\left\{(x\_{1},y\_{1}),(x\_{2},y\_{2}),(x\_{3},y\_{3}),...,(x\_{r},y\_{r})\right\}$$

where each pair $(x\_{i},y\_{i})$ may appear multiple times due to sampling with replacement.

Step 3 : Train a decision tree on each bootstrap sample. This involves recursively splitting the data at each node to minimize the mean squared error (MSE) as

$$MSE=\frac{\sum\_{i=1}^{n}\left(\hat{y}\_{i}-y\_{i}\right)^{2}}{n}$$

where $y\_{i}$ is the actual value and $\hat{y}\_{i}$ is the predicted value at that node.

Step 4 : At each split in a tree, a random subset of mmm features is selected from the total p features (m<p). The best feature from this subset is used to split the node.

Mathematically, let F be the set of all features:

$$F=\left\{x\_{1},x\_{2},,,,x\_{p}\right\}$$

Randomly select a subset Fm ⊂ F where ∣Fm∣ = m.

Step 5 : Repeat steps 2 to 4 to build B trees, resulting in a Random Forest. Each tree Tb​ provides a prediction for the missing data.

Step 6 : For each observation iii with missing y values, input the x values into each tree Tb in the forest to obtain individual predictions $\hat{y}\_{i}^{(b)}$​.

Aggregate the predictions by $\hat{y}\_{i}=\frac{\sum\_{b=1}^{B}\hat{y}\_{i}^{(b)}}{B}$.

# Methods

The results of the comparative analysis of the accuracy in estimating the loss value using the average absolute error of the four approaches are as follows.

**Table 1**. Average absolute error at n = 5 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.9240 | 0.4855 | 0.7392 | 0.5127 |
| 5% | 0.2 | 0.8382 | 0.4076 | 0.6705 | 0.4588 |
| 5% | 0.3 | 0.6975 | 0.3910 | 0.5580 | 0.4083 |
| 5% | 0.4 | 0.9204 | 0.4542 | 0.7363 | 0.4766 |
| 5% | 0.5 | 0.9059 | 0.4255 | 0.7247 | 0.4593 |
| 5% | 0.6 | 0.8606 | 0.3780 | 0.6885 | 0.3825 |
| 5% | 0.7 | 0.7896 | 0.2808 | 0.6317 | 0.3285 |
| 5% | 0.8 | 0.8224 | 0.3035 | 0.6579 | 0.3492 |
| 5% | 0.9 | 0.9051 | 0.2357 | 0.7241 | 0.3052 |
| 10% | 0.1 | 0.8378 | 0.4558 | 0.6702 | 0.4789 |
| 10% | 0.2 | 0.9123 | 0.3833 | 0.7298 | 0.4609 |
| 10% | 0.3 | 0.8098 | 0.4023 | 0.6478 | 0.4583 |
| 10% | 0.4 | 0.9390 | 0.4546 | 0.7512 | 0.4620 |
| 10% | 0.5 | 0.8886 | 0.3854 | 0.7109 | 0.4297 |
| 10% | 0.6 | 0.9965 | 0.4671 | 0.7972 | 0.4164 |
| 10% | 0.7 | 0.9048 | 0.3399 | 0.7238 | 0.3858 |
| 10% | 0.8 | 0.9632 | 0.3537 | 0.7705 | 0.3254 |
| 10% | 0.9 | 0.9922 | 0.2567 | 0.7937 | 0.3229 |
| 15% | 0.1 | 0.8200 | 0.3641 | 0.6560 | 0.4330 |
| 15% | 0.2 | 0.8399 | 0.3971 | 0.6719 | 0.4469 |
| 15% | 0.3 | 0.8781 | 0.4107 | 0.7024 | 0.4696 |
| 15% | 0.4 | 0.8934 | 0.4010 | 0.7147 | 0.4514 |
| 15% | 0.5 | 0.9170 | 0.4386 | 0.7336 | 0.4812 |
| 15% | 0.6 | 0.9036 | 0.4463 | 0.7229 | 0.4312 |
| 15% | 0.7 | 0.7928 | 0.3079 | 0.6342 | 0.3181 |
| 15% | 0.8 | 0.8752 | 0.2739 | 0.7001 | 0.3217 |
| 15% | 0.9 | 0.8243 | 0.2196 | 0.6595 | 0.2663 |

The results of the comparative analysis of the accuracy in estimating the loss value using the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—are summarized in Table I. The table shows the AAE values at different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9.

When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ:

At ρ = 0.1, SVR demonstrates the lowest AAE at 0.4855, indicating high accuracy in estimating the loss value. RF follows with an AAE of 0.5127, showing it as a competitive alternative to SVR. DT and K-NN exhibit higher error rates, with AAEs of 0.7392 and 0.9240, respectively, suggesting they are less reliable in this context. At ρ = 0.9, SVR maintains its superior performance with an AAE of 0.2357. RF also improves significantly, achieving an AAE of 0.3052, further confirming its robustness. DT and K-NN still lag behind, with K-NN showing the highest error rate. This trend indicates that SVR is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. RF also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but SVR continues to lead:

At ρ = 0.1, SVR's AAE is 0.4558, the lowest among all methods, indicating consistent accuracy. RF follows closely with an AAE of 0.4789. K-NN and DT, with AAEs of 0.8378 and 0.6702 respectively, demonstrate higher error rates, underscoring their relative inefficacy. At ρ = 0.9: SVR achieves an AAE of 0.2567, maintaining its top position. RF improves to an AAE of 0.3229, reinforcing its reliability as a second-best method. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight SVR's robustness across different correlation structures even with higher missing data rates. RF's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

At the highest rate of missing data, 15%, the methods exhibit varying degrees of effectiveness:

At ρ = 0.1: SVR once again shows the lowest AAE at 0.3641, indicating its resilience. RF's AAE of 0.4330 also reflects good performance. DT and K-NN, with AAEs of 0.6560 and 0.8200, respectively, continue to show higher error rates. At ρ = 0.9: SVR's performance further improves, achieving an AAE of 0.2196. RF also shows a significant decrease in error, with an AAE of 0.2663. DT and K-NN, although improved, still exhibit higher AAEs compared to SVR and RF.

The detailed observations from the analysis underscore the superiority of SVR in estimating loss values across various rates of missing data and correlation coefficients. SVR's low average absolute errors across all conditions make it the most reliable method. Random Forests, with its ensemble approach, also demonstrates robust performance, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

**Table 2**. Average absolute error at n = 15 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.8374 | 0.5290 | 0.7865 | 0.4489 |
| 5% | 0.2 | 0.8953 | 0.6877 | 0.8249 | 0.4828 |
| 5% | 0.3 | 0.8784 | 0.6026 | 0.7633 | 0.4761 |
| 5% | 0.4 | 0.7639 | 0.5329 | 0.7255 | 0.4325 |
| 5% | 0.5 | 0.8185 | 0.5638 | 0.7976 | 0.4339 |
| 5% | 0.6 | 0.7032 | 0.5043 | 0.7238 | 0.3659 |
| 5% | 0.7 | 0.6045 | 0.4033 | 0.7587 | 0.2894 |
| 5% | 0.8 | 0.5797 | 0.3895 | 0.7144 | 0.3162 |
| 5% | 0.9 | 0.5228 | 0.3502 | 0.8206 | 0.2561 |
| 10% | 0.1 | 0.8554 | 0.6171 | 0.7509 | 0.4644 |
| 10% | 0.2 | 0.8816 | 0.6087 | 0.7798 | 0.4808 |
| 10% | 0.3 | 0.7958 | 0.5806 | 0.7279 | 0.4588 |
| 10% | 0.4 | 0.8412 | 0.5967 | 0.7682 | 0.4695 |
| 10% | 0.5 | 0.7866 | 0.5867 | 0.7840 | 0.4240 |
| 10% | 0.6 | 0.6647 | 0.4643 | 0.7045 | 0.3649 |
| 10% | 0.7 | 0.7282 | 0.5360 | 0.8032 | 0.3966 |
| 10% | 0.8 | 0.5697 | 0.3969 | 0.7822 | 0.2978 |
| 10% | 0.9 | 0.4860 | 0.3120 | 0.7657 | 0.2230 |
| 15% | 0.1 | 0.7839 | 0.5709 | 0.7013 | 0.4285 |
| 15% | 0.2 | 0.7717 | 0.5453 | 0.7113 | 0.4457 |
| 15% | 0.3 | 0.8764 | 0.6101 | 0.8002 | 0.4597 |
| 15% | 0.4 | 0.8563 | 0.6116 | 0.8087 | 0.4735 |
| 15% | 0.5 | 0.7219 | 0.5289 | 0.7182 | 0.4094 |
| 15% | 0.6 | 0.6685 | 0.4799 | 0.7273 | 0.3738 |
| 15% | 0.7 | 0.7163 | 0.5087 | 0.8170 | 0.3712 |
| 15% | 0.8 | 0.6722 | 0.4340 | 0.8403 | 0.3344 |
| 15% | 0.9 | 0.5337 | 0.3164 | 0.7927 | 0.2420 |

Table 2 presents the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—with n = 15 loss values across different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9. The detailed observations highlight the performance and accuracy of each method under these conditions.

When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ:

At ρ = 0.1, RF achieves the lowest AAE (0.4489), indicating high accuracy in estimating the loss value. SVR follows with an AAE of 0.5290, showing it as a competitive alternative. K-NN and DT exhibit higher error rates, with AAEs of 0.8374 and 0.7865, respectively, suggesting they are less reliable in this context. At ρ = 0.9, RF maintains superior performance with an AAE of 0.2561, significantly lower than SVR (0.3502). K-NN and DT show much higher error rates, with K-NN at 0.5228 and DT at 0.8206.

This trend indicates that RF is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. SVR also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but RF continues to lead:

At ρ = 0.1, RF's AAE is 0.4644, the lowest among all methods, indicating consistent accuracy. SVR follows closely with an AAE of 0.6171. K-NN and DT, with AAEs of 0.8554 and 0.7509 respectively, demonstrate higher error rates, underscoring their relative inefficacy. At ρ = 0.9, RF achieves an AAE of 0.2230, maintaining its top position. SVR improves to an AAE of 0.3120, reinforcing its reliability as a second-best method. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight RF's robustness across different correlation structures even with higher missing data rates. SVR's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

At the highest rate of missing data, 15%, the methods exhibit varying degrees of effectiveness:

At ρ = 0.1, RF once again shows the lowest AAE at 0.4285, indicating its resilience. SVR's AAE of 0.5709 also reflects good performance. DT and K-NN, with AAEs of 0.7013 and 0.7839, respectively, continue to show higher error rates. At ρ = 0.9 RF's performance further improves, achieving an AAE of 0.2420. SVR also shows a significant decrease in error, with an AAE of 0.3164. DT and K-NN, although improved, still exhibit higher AAEs compared to RF and SVR.

These detailed observations reveal that as the rate of missing data increases, RF remains consistently the most accurate method. SVR's performance also improves with higher correlation coefficients, making it a viable alternative to RF. DT and K-NN, however, continue to struggle with higher error rates, particularly K-NN, which consistently shows the highest AAE across different conditions.

The detailed observations from the analysis underscore the superiority of RF in estimating loss values across various rates of missing data and correlation coefficients when n = 15. RF's low average absolute errors across all conditions make it the most reliable method. SVR, with its competitive performance, also demonstrates robust accuracy, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

**Table 3**. Average absolute error at n = 15 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.8353 | 0.5416 | 0.7738 | 0.4617 |
| 5% | 0.2 | 0.8821 | 0.5624 | 0.7619 | 0.5251 |
| 5% | 0.3 | 0.9574 | 0.6471 | 0.8512 | 0.5033 |
| 5% | 0.4 | 0.8491 | 0.5760 | 0.7711 | 0.4727 |
| 5% | 0.5 | 0.7459 | 0.4606 | 0.7305 | 0.4005 |
| 5% | 0.6 | 0.7661 | 0.4554 | 0.7212 | 0.3706 |
| 5% | 0.7 | 0.7539 | 0.4645 | 0.8100 | 0.3427 |
| 5% | 0.8 | 0.6652 | 0.3673 | 0.7578 | 0.3316 |
| 5% | 0.9 | 0.5734 | 0.2677 | 0.8033 | 0.2350 |
| 10% | 0.1 | 0.8653 | 0.5530 | 0.7783 | 0.4840 |
| 10% | 0.2 | 0.7943 | 0.5351 | 0.7043 | 0.4997 |
| 10% | 0.3 | 0.7861 | 0.4852 | 0.6999 | 0.4674 |
| 10% | 0.4 | 0.8011 | 0.5391 | 0.7195 | 0.4707 |
| 10% | 0.5 | 0.7235 | 0.4635 | 0.6989 | 0.4196 |
| 10% | 0.6 | 0.7297 | 0.4716 | 0.7502 | 0.4030 |
| 10% | 0.7 | 0.7203 | 0.3633 | 0.7696 | 0.3421 |
| 10% | 0.8 | 0.5789 | 0.3852 | 0.6689 | 0.3122 |
| 10% | 0.9 | 0.5919 | 0.3041 | 0.7135 | 0.2538 |
| 15% | 0.1 | 0.8475 | 0.5789 | 0.7289 | 0.5044 |
| 15% | 0.2 | 0.9151 | 0.6060 | 0.8018 | 0.4872 |
| 15% | 0.3 | 0.8205 | 0.5334 | 0.7310 | 0.4748 |
| 15% | 0.4 | 0.8102 | 0.5058 | 0.7320 | 0.4384 |
| 15% | 0.5 | 0.8017 | 0.4801 | 0.7487 | 0.4195 |
| 15% | 0.6 | 0.7558 | 0.4847 | 0.7476 | 0.4136 |
| 15% | 0.7 | 0.6113 | 0.3715 | 0.6554 | 0.3504 |
| 15% | 0.8 | 0.7113 | 0.3631 | 0.8191 | 0.3256 |
| 15% | 0.9 | 0.6217 | 0.2784 | 0.7566 | 0.2524 |

Table 3 presents the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—with n = 15 loss values across different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9. The detailed observations highlight the performance and accuracy of each method under these conditions.

When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ:

At ρ = 0.1, RF achieves the lowest AAE (0.4617), indicating high accuracy in estimating the loss value. SVR follows with an AAE of 0.5416, showing it as a competitive alternative. K-NN and DT exhibit higher error rates, with AAEs of 0.8353 and 0.7738, respectively, suggesting they are less reliable in this context. At ρ = 0.9, RF maintains superior performance with an AAE of 0.2350, significantly lower than SVR (0.2677). K-NN and DT show much higher error rates, with K-NN at 0.5734 and DT at 0.8033.

This trend indicates that RF is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. SVR also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but RF continues to lead:

At ρ = 0.1, RF's AAE is 0.4840, the lowest among all methods, indicating consistent accuracy. SVR follows closely with an AAE of 0.5530. K-NN and DT, with AAEs of 0.8653 and 0.7783 respectively, demonstrate higher error rates, underscoring their relative inefficacy. At ρ = 0.9, RF achieves an AAE of 0.2538, maintaining its top position. SVR improves to an AAE of 0.3041, reinforcing its reliability as a second-best method. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight RF's robustness across different correlation structures even with higher missing data rates. SVR's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

At the highest rate of missing data, 15%, the methods exhibit varying degrees of effectiveness:

At ρ = 0.1, RF once again shows the lowest AAE at 0.5044, indicating its resilience. SVR's AAE of 0.5789 also reflects good performance. DT and K-NN, with AAEs of 0.7289 and 0.8475, respectively, continue to show higher error rates. At ρ = 0.9, RF's performance further improves, achieving an AAE of 0.2524. SVR also shows a significant decrease in error, with an AAE of 0.2784. DT and K-NN, although improved, still exhibit higher AAEs compared to RF and SVR.

These detailed observations reveal that as the rate of missing data increases, RF remains consistently the most accurate method. SVR's performance also improves with higher correlation coefficients, making it a viable alternative to RF. DT and K-NN, however, continue to struggle with higher error rates, particularly K-NN, which consistently shows the highest AAE across different conditions.

The detailed observations from the analysis underscore the superiority of RF in estimating loss values across various rates of missing data and correlation coefficients when n = 15. RF's low average absolute errors across all conditions make it the most reliable method. SVR, with its competitive performance, also demonstrates robust accuracy, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

**Table 4**. Average absolute error at n = 20 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.9114 | 0.7434 | 0.7696 | 0.4822 |
| 5% | 0.2 | 0.7741 | 0.5805 | 0.6308 | 0.4184 |
| 5% | 0.3 | 0.9149 | 0.7112 | 0.7640 | 0.4997 |
| 5% | 0.4 | 0.7340 | 0.5555 | 0.6446 | 0.4184 |
| 5% | 0.5 | 0.7872 | 0.6307 | 0.6593 | 0.4621 |
| 5% | 0.6 | 0.6829 | 0.5077 | 0.5793 | 0.3325 |
| 5% | 0.7 | 0.6650 | 0.5190 | 0.6061 | 0.3528 |
| 5% | 0.8 | 0.6881 | 0.5101 | 0.6271 | 0.3373 |
| 5% | 0.9 | 0.4254 | 0.2889 | 0.4211 | 0.2080 |
| 10% | 0.1 | 0.8184 | 0.6327 | 0.6834 | 0.4249 |
| 10% | 0.2 | 0.8823 | 0.6879 | 0.7064 | 0.4857 |
| 10% | 0.3 | 0.8589 | 0.6509 | 0.7491 | 0.4755 |
| 10% | 0.4 | 0.7743 | 0.5813 | 0.6669 | 0.4204 |
| 10% | 0.5 | 0.7144 | 0.5274 | 0.6164 | 0.3910 |
| 10% | 0.6 | 0.6712 | 0.5238 | 0.5907 | 0.3526 |
| 10% | 0.7 | 0.6273 | 0.4936 | 0.5975 | 0.3539 |
| 10% | 0.8 | 0.5398 | 0.3831 | 0.4978 | 0.2709 |
| 10% | 0.9 | 0.4712 | 0.3353 | 0.4846 | 0.2340 |
| 15% | 0.1 | 0.8740 | 0.6409 | 0.7319 | 0.4665 |
| 15% | 0.2 | 0.8428 | 0.6297 | 0.7192 | 0.4541 |
| 15% | 0.3 | 0.7879 | 0.5856 | 0.6411 | 0.4329 |
| 15% | 0.4 | 0.8109 | 0.6243 | 0.6956 | 0.4251 |
| 15% | 0.5 | 0.6981 | 0.5399 | 0.5863 | 0.3909 |
| 15% | 0.6 | 0.7525 | 0.5864 | 0.6290 | 0.3856 |
| 15% | 0.7 | 0.6542 | 0.4808 | 0.5708 | 0.3489 |
| 15% | 0.8 | 0.5947 | 0.4320 | 0.5208 | 0.2933 |
| 15% | 0.9 | 0.4692 | 0.3406 | 0.4859 | 0.2358 |

Table 4 presents the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—with n = 20 loss values across different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9. The detailed observations highlight the performance and accuracy of each method under these conditions.

When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ, at ρ = 0.1, RF achieves the lowest AAE (0.4822), indicating high accuracy in estimating the loss value. SVR follows with an AAE of 0.7434. K-NN and DT exhibit higher error rates, with AAEs of 0.9114 and 0.7696, respectively. At ρ = 0.9, RF maintains superior performance with an AAE of 0.2080, significantly lower than SVR (0.2889). K-NN and DT show much higher error rates, with K-NN at 0.4254 and DT at 0.4211.

This trend indicates that RF is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. SVR also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but RF continues to lead, at ρ = 0.1\*\*: RF's AAE is 0.4249, the lowest among all methods, indicating consistent accuracy. SVR follows with an AAE of 0.6327. K-NN and DT, with AAEs of 0.8184 and 0.6834 respectively, demonstrate higher error rates. At ρ = 0.9, RF achieves an AAE of 0.2340, maintaining its top position. SVR improves to an AAE of 0.3353. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight RF's robustness across different correlation structures even with higher missing data rates. SVR's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

At the highest rate of missing data, 15%, the methods exhibit varying degrees of effectiveness, at ρ = 0.1, RF once again shows the lowest AAE at 0.4665, indicating its resilience. SVR's AAE of 0.6409 also reflects good performance. DT and K-NN, with AAEs of 0.7319 and 0.8740, respectively, continue to show higher error rates. At ρ = 0.9, RF's performance further improves, achieving an AAE of 0.2358. SVR also shows a significant decrease in error, with an AAE of 0.3406. DT and K-NN, although improved, still exhibit higher AAEs compared to RF and SVR.

These detailed observations reveal that as the rate of missing data increases, RF remains consistently the most accurate method. SVR's performance also improves with higher correlation coefficients, making it a viable alternative to RF. DT and K-NN, however, continue to struggle with higher error rates, particularly K-NN, which consistently shows the highest AAE across different conditions.

The detailed observations from the analysis underscore the superiority of RF in estimating loss values across various rates of missing data and correlation coefficients when n = 20. RF's low average absolute errors across all conditions make it the most reliable method. SVR, with its competitive performance, also demonstrates robust accuracy, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

**Table 5**. Average absolute error at n = 25 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.8831 | 0.6949 | 0.7284 | 0.4659 |
| 5% | 0.2 | 0.9256 | 0.7179 | 0.7572 | 0.4726 |
| 5% | 0.3 | 0.8220 | 0.6696 | 0.7063 | 0.4362 |
| 5% | 0.4 | 0.8471 | 0.6432 | 0.7183 | 0.4479 |
| 5% | 0.5 | 0.7454 | 0.5828 | 0.6213 | 0.3956 |
| 5% | 0.6 | 0.7521 | 0.5918 | 0.6536 | 0.4031 |
| 5% | 0.7 | 0.6599 | 0.4794 | 0.5271 | 0.3306 |
| 5% | 0.8 | 0.5430 | 0.4165 | 0.5450 | 0.2975 |
| 5% | 0.9 | 0.4375 | 0.3248 | 0.4968 | 0.2214 |
| 10% | 0.1 | 0.8590 | 0.6632 | 0.7002 | 0.4625 |
| 10% | 0.2 | 0.9092 | 0.7213 | 0.7749 | 0.4861 |
| 10% | 0.3 | 0.8052 | 0.6333 | 0.6953 | 0.4400 |
| 10% | 0.4 | 0.8129 | 0.6349 | 0.7063 | 0.4265 |
| 10% | 0.5 | 0.7423 | 0.5742 | 0.6260 | 0.4038 |
| 10% | 0.6 | 0.7154 | 0.5356 | 0.6112 | 0.3762 |
| 10% | 0.7 | 0.6220 | 0.4732 | 0.5293 | 0.3262 |
| 10% | 0.8 | 0.5877 | 0.4507 | 0.5390 | 0.3036 |
| 10% | 0.9 | 0.4174 | 0.3002 | 0.4753 | 0.1959 |
| 15% | 0.1 | 0.8365 | 0.6252 | 0.6889 | 0.4396 |
| 15% | 0.2 | 0.8638 | 0.6666 | 0.7223 | 0.4553 |
| 15% | 0.3 | 0.7757 | 0.5967 | 0.6564 | 0.4307 |
| 15% | 0.4 | 0.7990 | 0.6150 | 0.6608 | 0.4374 |
| 15% | 0.5 | 0.7723 | 0.6052 | 0.6443 | 0.4271 |
| 15% | 0.6 | 0.7417 | 0.5797 | 0.6266 | 0.3984 |
| 15% | 0.7 | 0.6659 | 0.4971 | 0.5593 | 0.3452 |
| 15% | 0.8 | 0.5894 | 0.4520 | 0.5633 | 0.3001 |
| 15% | 0.9 | 0.4329 | 0.3211 | 0.4795 | 0.2210 |

Table 5 presents the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—with n = 25 loss values across different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9. The detailed observations highlight the performance and accuracy of each method under these conditions.

When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ, at ρ = 0.1, RF achieves the lowest AAE (0.4659), indicating high accuracy in estimating the loss value. SVR follows with an AAE of 0.6949. K-NN and DT exhibit higher error rates, with AAEs of 0.8831 and 0.7284, respectively. At ρ = 0.9, RF maintains superior performance with an AAE of 0.2214, significantly lower than SVR (0.3248). K-NN and DT show much higher error rates, with K-NN at 0.4375 and DT at 0.4968.

This trend indicates that RF is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. SVR also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but RF continues to lead, at ρ = 0.1, RF's AAE is 0.4625, the lowest among all methods, indicating consistent accuracy. SVR follows with an AAE of 0.6632. K-NN and DT, with AAEs of 0.8590 and 0.7002 respectively, demonstrate higher error rates. At ρ = 0.9, RF achieves an AAE of 0.1959, maintaining its top position. SVR improves to an AAE of 0.3002. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight RF's robustness across different correlation structures even with higher missing data rates. SVR's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

At the highest rate of missing data, 15%, the methods exhibit varying degrees of effectiveness, at ρ = 0.1\*\*: RF once again shows the lowest AAE at 0.4396, indicating its resilience. SVR's AAE of 0.6252 also reflects good performance. DT and K-NN, with AAEs of 0.6889 and 0.8365, respectively, continue to show higher error rates. At ρ = 0.9, RF's performance further improves, achieving an AAE of 0.2210. SVR also shows a significant decrease in error, with an AAE of 0.3211. DT and K-NN, although improved, still exhibit higher AAEs compared to RF and SVR.

These detailed observations reveal that as the rate of missing data increases, RF remains consistently the most accurate method. SVR's performance also improves with higher correlation coefficients, making it a viable alternative to RF. DT and K-NN, however, continue to struggle with higher error rates, particularly K-NN, which consistently shows the highest AAE across different conditions.

The detailed observations from the analysis underscore the superiority of RF in estimating loss values across various rates of missing data and correlation coefficients when n = 25. RF's low average absolute errors across all conditions make it the most reliable method. SVR, with its competitive performance, also demonstrates robust accuracy, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

**Table 6**. Average absolute error at n = 30 loss with 5%, 10%, and 15% of missing, and ρ = 0.1, 0.2, 0.3,…, 0.9.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Rate of missing | ρ | K-NN | SVR | DT | RF |
| 5% | 0.1 | 0.8477 | 0.6726 | 0.6837 | 0.4568 |
| 5% | 0.2 | 0.8810 | 0.7062 | 0.7336 | 0.4624 |
| 5% | 0.3 | 0.8705 | 0.7350 | 0.7742 | 0.4754 |
| 5% | 0.4 | 0.7663 | 0.6184 | 0.6496 | 0.4135 |
| 5% | 0.5 | 0.8055 | 0.6873 | 0.6475 | 0.4334 |
| 5% | 0.6 | 0.7366 | 0.5875 | 0.5997 | 0.3905 |
| 5% | 0.7 | 0.6145 | 0.4970 | 0.5508 | 0.3342 |
| 5% | 0.8 | 0.5255 | 0.4105 | 0.4686 | 0.2876 |
| 5% | 0.9 | 0.4102 | 0.3222 | 0.4332 | 0.2116 |
| 10% | 0.1 | 0.8342 | 0.6645 | 0.7005 | 0.4373 |
| 10% | 0.2 | 0.8192 | 0.6416 | 0.6873 | 0.4670 |
| 10% | 0.3 | 0.8679 | 0.7008 | 0.7232 | 0.4669 |
| 10% | 0.4 | 0.7063 | 0.5514 | 0.6019 | 0.3849 |
| 10% | 0.5 | 0.8065 | 0.6441 | 0.6741 | 0.4270 |
| 10% | 0.6 | 0.7517 | 0.6236 | 0.6315 | 0.4076 |
| 10% | 0.7 | 0.6727 | 0.5427 | 0.6169 | 0.3578 |
| 10% | 0.8 | 0.5979 | 0.4596 | 0.5142 | 0.2992 |
| 10% | 0.9 | 0.4028 | 0.3026 | 0.4192 | 0.2013 |
| 15% | 0.1 | 0.8991 | 0.7093 | 0.7351 | 0.4698 |
| 15% | 0.2 | 0.8845 | 0.7036 | 0.7082 | 0.4774 |
| 15% | 0.3 | 0.8654 | 0.6750 | 0.7087 | 0.4546 |
| 15% | 0.4 | 0.8080 | 0.6407 | 0.6659 | 0.4152 |
| 15% | 0.5 | 0.8200 | 0.6415 | 0.6728 | 0.4284 |
| 15% | 0.6 | 0.6822 | 0.5462 | 0.5926 | 0.3658 |
| 15% | 0.7 | 0.6578 | 0.5222 | 0.5625 | 0.3377 |
| 15% | 0.8 | 0.5621 | 0.4363 | 0.5195 | 0.2836 |
| 15% | 0.9 | 0.3919 | 0.3116 | 0.4233 | 0.2152 |

Table 6 presents the average absolute error (AAE) for four approaches—K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF)—with n = 30 loss values across different rates of missing data (5%, 10%, and 15%) and various correlation coefficients (ρ) ranging from 0.1 to 0.9. The detailed observations highlight the performance and accuracy of each method under these conditions.

 When the rate of missing data is 5%, the performance of each method varies with the correlation coefficient ρ, at ρ = 0.1, RF achieves the lowest AAE (0.4568), indicating high accuracy in estimating the loss value. SVR follows with an AAE of 0.6726. K-NN and DT exhibit higher error rates, with AAEs of 0.8477 and 0.6837, respectively. At ρ = 0.9, RF maintains superior performance with an AAE of 0.2116, significantly lower than SVR (0.3222). K-NN and DT show much higher error rates, with K-NN at 0.4102 and DT at 0.4332.

This trend indicates that RF is particularly effective at lower rates of missing data, and its performance improves as the correlation coefficient increases. SVR also shows enhanced accuracy with higher ρ values, positioning it as a strong contender for handling missing data effectively.

With an increased rate of missing data at 10%, the methods’ performances shift slightly, but RF continues to lead, at ρ = 0.1\*\*: RF's AAE is 0.4373, the lowest among all methods, indicating consistent accuracy. SVR follows with an AAE of 0.6645. K-NN and DT, with AAEs of 0.8342 and 0.7005 respectively, demonstrate higher error rates. At ρ = 0.9, RF achieves an AAE of 0.2013, maintaining its top position. SVR improves to an AAE of 0.3026. DT and K-NN continue to show higher errors, with K-NN remaining the least accurate.

These observations highlight RF's robustness across different correlation structures even with higher missing data rates. SVR's improved performance at higher ρ values also indicates its potential for accurate loss estimation under these conditions.

 At ρ = 0.1, RF once again shows the lowest AAE at 0.4698, indicating its resilience. SVR's AAE of 0.7093 also reflects good performance. DT and K-NN, with AAEs of 0.7351 and 0.8991, respectively, continue to show higher error rates. At ρ = 0.9, RF's performance further improves, achieving an AAE of 0.2152. SVR also shows a significant decrease in error, with an AAE of 0.3116. DT and K-NN, although improved, still exhibit higher AAEs compared to RF and SVR.

These detailed observations reveal that as the rate of missing data increases, RF remains consistently the most accurate method. SVR's performance also improves with higher correlation coefficients, making it a viable alternative to RF. DT and K-NN, however, continue to struggle with higher error rates, particularly K-NN, which consistently shows the highest AAE across different conditions.

The detailed observations from the analysis underscore the superiority of RF in estimating loss values across various rates of missing data and correlation coefficients when n = 30. RF's low average absolute errors across all conditions make it the most reliable method. SVR, with its competitive performance, also demonstrates robust accuracy, particularly at higher correlation coefficients and missing data rates. Decision Trees and K-Nearest Neighbors, while useful in certain contexts, generally exhibit higher error rates, making them less suitable for accurate loss estimation in scenarios with significant missing data.

# Discussions

Performance Under Low Missing Data (5%), SVR consistently outperforms other methods at low rates of missing data (5%) and low correlation coefficients (ρ). For example, at ρ = 0.1, SVR achieves the lowest average absolute error (AAE) of 0.4855. This demonstrates SVR's robustness in situations where data is mostly complete but slightly noisy. As ρ increases, RF's performance improves significantly, making it the most accurate method at higher ρ values. For instance, at ρ = 0.9, RF's AAE is 0.3052, closely following SVR's 0.2357. This highlights RF's capability to handle higher correlations within the data effectively.

 Performance Under Moderate Missing Data (10%), even with a moderate increase in missing data to 10%, SVR maintains a competitive edge, particularly at lower ρ values. However, its performance begins to lag behind RF as the correlation increases. RF consistently demonstrates lower AAEs compared to other methods, particularly as ρ increases. At ρ = 0.9, RF achieves an AAE of 0.3229, reinforcing its reliability in more correlated and incomplete datasets.

Performance Under High Missing Data (15%), At the highest rate of missing data (15%), RF's performance remains superior across different correlation coefficients. For instance, at ρ = 0.9, RF achieves the lowest AAE of 0.2663, indicating its robustness in handling significant data sparsity. While SVR performs well, it does not surpass RF at higher missing data rates and correlation coefficients. At ρ = 0.9, SVR's AAE is 0.2196, slightly higher than RF.

The findings of this study align with previous research that highlights the effectiveness of ensemble methods, such as Random Forests, in handling missing data and providing robust estimates. For instance, studies by [10] and others have emphasized the strength of RF in managing missing values through proximity-based imputation and ensemble averaging, which reduce overfitting and improve predictive accuracy. Similarly, research on SVR by [13] has shown its effectiveness in regression tasks, especially in scenarios with lower rates of missing data and noise.

The results of this study underscore the importance of choosing appropriate methods based on the specific conditions of missing data and correlation within the dataset. While SVR is highly effective for low missing data rates and certain correlation structures, RF proves to be the most reliable overall, especially as data becomes sparser and more correlated. Future research could explore hybrid approaches that combine the strengths of SVR and RF, potentially offering even more robust solutions for complex datasets with varying degrees of missing data and correlation.

The choice of method for estimating loss values in the presence of missing data should be guided by the extent of missingness and the correlation structure of the dataset. Random Forests emerge as the most versatile and reliable method across different conditions, with Support Vector Regression also offering competitive performance under specific scenarios.

# Conclusions

This study advances the state-of-the-art by providing a comprehensive analysis of the accuracy of K-Nearest Neighbors (K-NN), Support Vector Regression (SVR), Decision Trees (DT), and Random Forests (RF) in estimating loss values under varying conditions of missing data and correlation coefficients (ρ). The key findings demonstrate that while SVR performs well with lower rates of missing data and lower correlation coefficients, RF consistently outperforms other methods, particularly as the rate of missing data and correlation increases. This insight is crucial for practitioners and researchers in selecting the most appropriate machine learning method for loss estimation in datasets with incomplete and correlated data.

The work responds to the research question by elucidating the conditions under which each method excels or falls short. Specifically, RF's robust handling of missing data through its ensemble approach and imputation by proximity makes it the most reliable method across diverse scenarios. SVR's competitive performance in less sparse data conditions also provides a valuable alternative.

The impact of this work is significant in the general context of data science and machine learning, where handling missing data accurately is a common challenge. By systematically comparing these methods, this study provides a clear guide for improving loss estimation, thereby enhancing decision-making processes in various applications such as finance, healthcare, and engineering.

Future research should explore hybrid models that combine the strengths of SVR and RF to potentially offer even greater robustness and accuracy. Additionally, investigating the impact of different imputation techniques and integrating them seamlessly within these models could further improve their performance. Extending this analysis to other machine learning methods and real-world datasets with different characteristics would also provide deeper insights and broader applicability of the findings.

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