# TriVec: Knowledge Graph Embeddings for Accurate and Efficient Link Prediction in Real World Application Scenarios

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Abstract. Knowledge graph embeddings models are widely used to provide scalable and efficient link prediction for knowledge graphs. They use different techniques to model embeddings interactions, where their tensor factorisation based versions are known to provide state-of-the-art results. In recent works, developments on factorisation based knowledge graph embedding models were mostly limited to enhancing the ComplEx and the DistMult models, as they can efficiently provide predictions within linear time and space complexity. The evaluation of these models was also limited to general knowledge benchmarks and it did not include any other applications in specialised domains. In this work, we aim to extend the works of the ComplEx and the DistMult models by proposing a new factorisation model, TriVec, which uses three part embeddings to model a combination of symmetric and asymmetric interactions between embeddings. We perform an empirical evaluation for the TriVec model compared to other tensor factorisation models on different training configurations (loss functions and regularisation terms), and we show that the TriVec model provides the state-of-the-art results in all configurations. In our experiments, we use standard benchmarking datasets (WN18, WN18RR, FB15k, FB15k-237, YAGO10) along with a new NELL based benchmarking dataset (NELL239) that we have developed. To complement the evaluation of our method on standard, but rather artificial datasets, we also present a more realistic benchmark based on the real-world problem of predicting effects of chemical-protein interactions. More specifically, we build a knowledge graph benchmark of chemicals, proteins and the effects of their interactions, and we desing an evaluation pipeline that uses knowledge graph embedding to predict new chemical-protein interactions and their effects. We then show by experimental evaluation that our model provides the best results in terms of the area under the ROC and precision recall curves in the prediction of the effects of chemical-protein interactions compared to other knowledge graph embedding models.

Keywords. Knowledge Graph Embedding, Link Prediction, Bioinformatics

# 1. Introduction

In recent years, knowledge graph embedding (KGE) models have witnessed rapid developments that have allowed them to excel in the task of link prediction for knowledge graphs [1]. They learn embeddings using different techniques like tensor factorisation,

latent distance similarity and convolutional filters in order to rank facts in the form of (subject, predicate, object) triples according to their factuality. In this context, their tensor factorisation based versions like the DistMult [2] and the ComplEx [3] models are known to provide state-of-the-art results within linear time and space complexity [1]. The scalable and efficient predictions achieved by these models have encouraged researchers to investigate advancing the DistMult and the ComplEx models by utilising different training objectives and regularisation terms [4,5].

In this work, our objective is to propose a new factorisation based knowledge graph embedding model that extends the works of the DistMult and the ComplEx models while preserving their linear time and space complexity. We achieve that by modifying two of their main components: the embedding representation, and the embedding interaction function.

While both the DistMult and the ComplEx models use the bilinear product of the subject, the predicate and the object embeddings as an embedding interaction function to encode knowledge facts, they represent their embeddings using different systems. The DistMult model uses real values to represent its embedding vectors, which leads to learning a symmetric representation of all predicates due to the symmetric nature of the product operator on real numbers. On the other hand, the ComplEx model represents embeddings using complex numbers, where each the embeddings of an entity or a relation is represented using two vectors (real and imaginary parts). The ComplEx model also represents entities in the object mode as the complex conjugate of their subject form [3]. This enables the ComplEx model to encode both symmetric and asymmetric predicates.

Since the embeddings of the ComplEx models are represented using two part embeddings (real and imaginary parts), their bilinear product (ComplEx's embedding interaction function) consists of different interaction components unlike the DisMult model with only one bilinear product component. Each of these components is a bilinear product of a combination of real and imaginary vectors of the subject, the predicate and the object embeddings, which gives the ComplEx model its ability to model asymmetric predicates.

In this work, we investigate both the embedding representation and the embedding interaction components of the ComplEx model, where we show that the ComplEx embedding interaction components are necessary but not sufficient to model asymmetric predicates. We also show that our proposed model, TriVec, can efficiently encode both symmetric and asymmetric predicates using simple embedding interaction components that rely on embeddings of three parts [6]. To assess our model compared to the ComplEx model, we carry experiments on both models using different training objectives and regularisation terms, where our results show that our new model, TriVec, provide equivalent or better results than the ComplEx model on all configurations. We also propose a new NELL [7] based benchmarking dataset that contains a small number of training, validation and testing facts that can be used to facilitate fast development of new knowledge graph embedding models.

Previous work on the evaluation of KGE models is mostly focused on general human knowledge benchmarks which are based on knowledge graphs such as YAGO, Freebase, WordNet, etc. However, an evaluation on a real world application on a specialised domain is largely missing, which means it is rather hard to assess the practical value of the

existing predictive models based on knowledge graphs. To address this gap, we present the problem of predicting the biological effects of chemical-protein interactions. This is a biological task motivated by a real world need [8] that we use to evaluate the accuracy of KGE models. We model the chemical-protein interactions and their effects as tensors, we then use KGE models to learn representation of chemicals, proteins and the biological effects in order to be able to predict new chemical, protein and effects combinations. We execute an evaluation pipeline where we compare between the predictive accuracy of our model and other KGE model. The results of our experiments show that our model outperforms other approaches in terms of the area under the ROC and precision recall curves.

## 2. Background and Related Works

Knowledge graph embedding models learn low rank vector representation *i.e.* embeddings for graph entities and relations. In the link prediction task, they learn embeddings in order to rank knowledge graph facts according to their factuality. The process of learning these embeddings consists of different phases. First, they initialise embeddings using random noise. These embeddings are then used to score a set of true and false facts, where a score of a fact is generated by computing the interaction between the fact's subject, predicate and object embeddings using a model dependent scoring function. Finally, embeddings are updated by a training loss that usually represents a min-max loss, where the objective is to maximise true facts scores and minimise false facts scores.

In this section we discuss scoring functions and training loss functions in state-of-theart knowledge graph embedding models. We define our notation as follows: for any given knowledge graph, *E* is the set of all entities, *R* is the set of all relations *i.e.* predicates,  $N_e$ and  $N_r$  are the numbers of entities and relations respectively, *T* is the set of all known true facts, *e* and *w* are matrices of sizes  $N_e \times K$  and  $N_r \times K$  respectively that represent entities and relations embeddings of rank *K*,  $\phi_{spo}$  is the score of the triple (s, p, o), and  $\mathcal{L}$  is the model's training loss.

## 2.1. Scoring Functions

Knowledge graph embedding models generate scores for facts using model dependent scoring functions that compute interactions between facts' components embeddings. These functions use different approaches to compute embeddings interactions like distance between embeddings [9], embedding factorisation [3] or embeddings convolutional filters [10].

In the following, we present these approaches and specify some examples of knowledge graph embedding models that use them.

• Distance-based embeddings interactions: The Translating Embedding model (TransE) [9] is one of the early models that use distance between embeddings to generate triple scores. It interprets triple's embeddings interactions as a linear translation of the subject to the object such that  $e_s + w_p = e_o$ , and generates a score for a triple as follows:

$$\phi_{spo}^{\text{TransE}} = \|e_s + w_p - e_o\|_{l1/l2},\tag{1}$$

where true facts have zero score and false facts have higher scores. This approach provides scalable and efficient embeddings learning as it has linear time and space complexity. However, it fails to provide efficient representation for interactions in one-to-many, many-to-many and many-to-one predicates as its design assumes one object per each subject-predicate combination.

• *Factorisation-based embedding interactions*: Interactions based on embedding factorisation provide better representation for predicates with high cardinality. They have been adopted in models like DistMult [2] and ComplEx [3]. The DistMult model uses the bilinear product of embeddings of the subject, the predicate, and the object as their interaction, and its scoring function is defined as follows:

$$\phi_{spo}^{\text{DistMult}} = \sum_{k=1}^{K} e_{s_k} w_{p_k} e_{o_k} \tag{2}$$

where  $e_{s_k}$  is the *k*-th component of subject entity *s* embedding vector  $e_s$ . DistMult achieved a significant improvement in accuracy in the task of link prediction over models like TransE. However, the symmetry of embedding scoring functions affects its predictive power on asymmetric predicates as it cannot capture the direction of the predicate. On the other hand, the ComplEx model uses embedding in a complex form to model data with asymmetry. It models embeddings interactions using the the product of complex embeddings, and its scores are defined as follows:

$$\phi_{spo}^{\text{ComplEx}} = \text{Re}(\sum_{k=1}^{K} e_{s_k} w_{p_k} \overline{e}_{o_k})$$

$$= \sum_{k=1}^{K} e_{s_k}^r w_{p_k}^r e_{o_k}^r + e_{s_k}^i w_{p_k}^r e_{o_k}^i$$

$$+ e_{s_k}^r w_{p_k}^i e_{o_k}^i - e_{s_k}^i w_{p_k}^j e_{o_k}^r$$
(3)

where Re(x) represents the real part of complex number x and all embeddings are in complex form such that  $e, w \in \mathbb{C}$ ,  $e^r$  and  $e^i$  are respectively the real and imaginary parts of e, and  $\overline{e}_o$  is the complex conjugate of the object embeddings  $e_o$  such that  $\overline{e}_o = e_o^r - ie_o^i$  and this introduces asymmetry to the scoring function. Using this notation, ComplEx can handle data with asymmetric predicates, and to keep scores in the real spaces it only uses the real part of embeddings product outcome. ComplEx preserves both linear time and linear space complexities as in TransE and DistMult, however, it surpasses their accuracies in the task of link prediction due to its ability to model a wider set of predicate types.

• Convolution-based embeddings interactions: Following the success of convolutional neural networks image processing tasks, models like R-GCN [11] and ConvE [10] utilized convolutional networks to learn knowledge graph embeddings. The R-GCN model learns entity embeddings using a combination of convolutional filters of its neighbours, where each predicate represent a convolution filter and each neighbour entity represents an input for the corresponding predicate filter. This approach is combined with the DistMult



Figure 1. Plots of different knowledge graph embedding loss functions and their training error slopes. Red lines represent the score/loss slopes of the false triples and blue lines represent the score/loss slopes of the true triples.

model to perform link prediction. Meanwhile, the ConvE model concatenates subject and predicate embeddings vectors into an image (a matrix form), then it uses a 2D convolutional pipeline to transform this matrix into a vector and computes its interaction with the object entity embeddings to generate a corresponding score as follows:

$$\phi_{spo}^{\text{ConvE}} = f(\operatorname{vec}(f([\overline{e_s}; \overline{w_p}] * \boldsymbol{\omega}))W)e_o \tag{4}$$

where  $\overline{e_s}$  and  $\overline{w_p}$  denotes a 2D reshaping of  $e_s$  and  $w_p$ ,  $\omega$  is a convolution filter, f denotes a non-linear function, vec(x) is a transformation function that reshape matrix x of size  $m \times n$  into a vector of size  $mn \times 1$ .

# 2.2. Loss Functions

The task of link prediction can generally be cast as a learning to rank problem where the object is to rank knowledge graph triples according to their factuality. Thus, knowledge graph embedding models traditionally use ranking loss approaches like pairwise and pointwise loss functions as in TransE and ComplEx respectively to model their training loss during the learning process. In these approaches a set of negative facts *i.e.* corruptions, is generated using a uniform random sample of entities to represent false facts, where training loss uses a min-max approach to maximise true facts scores and minimise false facts scores. Meanwhile, recent attempts considered using a multi-class loss to represent training error, where a triple (s, p, o) is divided into an input (s, p) and a corresponding class *o* and the objective is to assign class *o* to the (s, p) input.

In the following, we discuss these two approaches with examples from state-of-the-art knowledge graph embedding models.

• *Ranking loss functions*: Knowledge graph embedding models has adopted different pointwise and pairwise ranking losses like hinge loss and logistic loss to model their training loss. Hinge loss can be interpreted as a pointwise loss or a pairwise loss that minimises the scores of negative facts and maximise the scores of positive facts to reach a specific configurable value. This approach is used in HolE [12], and it is defined as:

$$\mathcal{L}_{\text{Hinge}} = \sum_{x \in X} [\lambda - l(x) \cdot f(x)]_+, \tag{5}$$

where l(x) = 1 if x is true and -1 otherwise and  $[c]_+$  is equal to max(c, 0). This effectively generates two different loss slopes for positive and negative scores as shown in Fig. 1.

The squared error loss can also be adopted as a pointwise ranking loss function. For example, the RESCAL [13] model uses the squared error to model its training loss with the objective of minimising the difference between model scores and their actual labels:

$$\mathcal{L}_{SE} = \frac{1}{2} \sum_{i=1}^{n} (f(x_i) - l(x_i))^2.$$
(6)

The optimal score for true and false facts is 1 and 0, respectively, as shown in Fig. 1. Also, the squared loss requires less training time since it does not require configurable training parameters, shrinking the search space of hyperparameters compared to other losses (*e.g.*, the margin parameter of the hinge loss).

The ComplEx [3] model uses a logistic loss, which is a smoother version of pointwise hinge loss without the margin requirement (*cf.* Fig. 1). Logistic loss uses a logistic function to minimise negative triples score and maximise positive triples score. This is similar to hinge loss, but uses a smoother linear loss slope defined as:

$$\mathcal{L}_{\text{logistic}_{P_t}} = \sum_{x \in T} \log(1 + \exp(-l(x) \cdot f(x))), \tag{7}$$

where l(x) is the true label of fact x that is equal to 1 for positive facts and is equal to -1 otherwise.

• *Multi-class loss approach*: ConvE model proposed a new binary cross entropy multiclass loss to model its training error. In this setting, the whole vocabulary of entities is used to train each positive fact that for a triple (s, p, o) all facts (s, p, o') with  $o' \in E$  and  $o' \neq o$ are considered false. Despite the extra computational cost of this approach, it allowed ConvE to generalise over a larger sample of negative assistances therefore surpassing other approaches in accuracy [10]. In a recent work, Lacroix et. al. [5] introduced a softmax regression loss to model the training error of the ComplEx model as a multi-class problem. In this approach, the objective for each triple (s, p, o) is to minimise the following losses:

$$\mathcal{L}_{spo}^{\log-\text{softmax}} = \mathcal{L}_{spo}^{o'} + \mathcal{L}_{spo}^{s'},$$

$$\mathcal{L}_{spo}^{o'} = -\phi_{spo} + \log(\sum_{o'} \exp(\phi_{spo'}))$$

$$\mathcal{L}_{spo}^{s'} = -\phi_{spo} + \log(\sum_{s'} \exp(\phi_{s'po}))$$
(8)

where  $s' \in E$ ,  $s' \neq s$ ,  $o' \in E$  and  $o' \neq o$ . This resembles a log-loss of the softmax value of the positive triple compared to all possible object and subject corruptions where the objective is to maximise positive facts scores and minimise all other scores. This approach achieved a significant improvement to the prediction accuracy of ComplEx model over all benchmark datasets [5].

Model	Definition	NEI	LL239	WN	18RR	FB237		
		MRR	H@10	MRR	H@10	MRR	H@10	
ComplEx	i1+i2+i3-i4	0.35	0.51	0.44	0.51	0.22	0.41	
ComplEx-V1	i1+i2+i3	0.34	0.51	0.45	0.52	0.22	0.40	
ComplEx-V2	i2+i3+i4	0.34	0.50	0.44	0.51	0.21	0.38	
ComplEx-V3	i1+i2-i4	0.34	0.51	0.45	0.52	0.22	0.40	
ComplEx-V4	i1+i3-i4	0.33	0.50	0.45	0.50	0.21	0.39	

 Table 1. A comparison between the ComplEx model and different variants of its scoring functions on standard benchmarking datasets

#### 2.3. Ranking Evaluation Metrics

Learning to rank models are evaluated using different ranking measures including Mean Average Precision (MAP), Normalised Discounted Cumulative Gain (NDCG), and Mean Reciprocal Rank (MRR). In this study, we only focus on the Mean Reciprocal Rank (MRR) since it is the main metric used in previous related works.

**Mean Reciprocal Rank (MRR).** The Reciprocal Rank (RR) is a statistical measure used to evaluate the response of ranking models depending on the rank of the first correct answer. The MRR is the average of the reciprocal ranks of results for different queries in Q. Formally, MRR is defined as:

$$\mathrm{MRR} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\mathcal{R}(\mathbf{x}_i, f)},$$

where  $x_i$  is the highest ranked relevant item for query  $q_i$ . Values of RR and MRR have a maximum of 1 for queries with true items ranked first, and get closer to 0 when the first true item is ranked in lower positions.

# 3. The TriVec Model

In this section, we motivate for the design decision of TriVec model, and we present its way to model embeddings interaction and training loss.

## 3.1. Motivation

Currently, models using factorisation-based knowledge graph embedding approaches like DistMult and ComplEx achieve state-of-the-art results across all benchmarking datasets [5]. In the DistMult model, embeddings interactions are modelled using a symmetric function that computes the product of embeddings of the subject, the predicate and the object. This approach was able to surpass other distance-based embedding techniques like TransE [2]. However, it failed to model facts with asymmetric predicate due to its design. The ComplEx model tackle this problem using a embeddings in the complex space where its embeddings interactions use the complex conjugate of object embeddings to break the symmetry of the interactions. This approach provided significant accuracy improvements over DistMult as it successfully models a wider range of predicates.



**Figure 2.** Visual explanation for the flow of the TriVec model's scoring function, where embedding interactions are represented by  $i_1 = e_s^3 w_p^3 e_o^1$ ,  $i_2 = e_s^2 w_p^2 e_o^2$ , and  $i_3 = e_s^1 w_p^1 e_o^3$ .

The ComplEx embeddings interaction function (defined in Sec. 2) can be redefined as a simple set of interactions of two part embeddings as follows:

$$\phi_{spo}^{\text{ComplEx}} = \sum_{k} i_1 + i_2 + i_3 - i_4 \tag{9}$$

where  $\sum_k$  is the sum of all embeddings components of index  $k = \{1, ..., K\}$ , and interactions  $i_1, i_2, i_3$  and  $i_4$  are defined as follows:

$$i_1 = e_s^1 w_p^1 e_o^1$$
,  $i_2 = e_s^2 w_p^1 e_o^2$ ,  $i_3 = e_s^1 w_p^2 e_o^2$ ,  $i_4 = e_s^2 w_p^2 e_o^1$ 

where  $e^1$  represents embeddings part 1, and  $e^2$  is part 2 (1  $\rightarrow$  real and 2  $\rightarrow$  imaginary). Following this notation, we can see that the ComplEx model is a set of two symmetric interaction  $i_1$  and  $i_2$  and two asymmetric interactions  $i_3$  and  $i_4$ . Furthermore, this encouraged us to investigate the effect of using other forms of combined symmetric and asymmetric interactions to model embeddings interactions in knowledge graph embeddings. We investigated different combination of interactions  $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$ , and we have found that by removing and/or changing the definition of one of these interactions (maintaining that the interactions use all triple components) will preserve similar or insignificantly different prediction accuracy across different benchmarking datasets (See Table 1). This led us to investigate other different forms of interactions that uses a combination of symmetric and asymmetric interactions where we found that using embeddings of three parts can lead to better predictive accuracy than the ComplEx and the DistMult models.

## 3.2. TriVec Embeddings Interactions

In the TriVec model, we represent each entity and relation using three embedding vectors such that the embedding of entity *i* is  $\{e_i^1, e_i^2, e_i^3\}$  and the embedding of relation *j* is  $\{w_j^1, w_j^2, w_j^3\}$  where  $e^m$  denotes the *m* part of the embeddings and where  $m \in 1, 2, 3$  is used to represent the three embeddings parts.

The TriVec model is a tensor factorisation based model, where its embeddings interaction function (scoring function) is defined as follows:

$$\phi_{spo}^{\text{TriPart}} = \sum_{k=1}^{K} e_{sk}^{1} w_{pk}^{1} e_{ok}^{3} + e_{sk}^{2} w_{pk}^{2} e_{ok}^{2} + e_{sk}^{3} w_{pk}^{3} e_{ok}^{1}$$
(10)

where k denotes the index of the embedding vector entries. The model uses a set of three interactions: one symmetric interaction:  $(e_s^2 w_p^2 e_o^2)$  and two asymmetric interactions:  $(e_s^1 w_p^1 e_o^3)$  and  $(e_s^3 w_p^3 e_o^1)$  as shown in Fig. 2. This approach models both symmetry and asymmetry in a simple form similar to the DistMult model where the DisMult model can be seen as a special case of the TriVec model if the first and third embeddings part are equivalent  $(e^1 = e^3)$ .

## 3.3. Training the TriVec Model

Trouillon et. al. [14] showed that despite the equivalence of HolE and ComplEx models' scoring functions, they produce different results as they use different loss functions. They concluded that the logistic loss version of ComplEx outperforms its hinge loss version. In addition, we have investigated different other ranking losses with the ComplEx model, and we have found that squared error loss can significantly enhance the performance of ComplEx on multiple benchmarking datasets.

The TriVec model performs its learning process using two different training loss configurations: the traditional ranking loss and the multi-class loss. In the ranking loss configuration, the TriVec model uses the squared error (Eq. 6) and the logistic loss (Eq. 7) to model its training error, where a grid search is performed to choose the optimal loss representation for each dataset. In the multi-class configuration, it uses the negative-log softmax loss (Eq. 8) with the nuclear 3-norm regularisation [5] which is defined as follows:

$$\mathcal{L}_{spo}^{\text{TriVec}} = -\phi_{spo} + \log(\sum_{o'} \exp(\phi_{spo'})) -\phi_{spo} + \log(\sum_{s'} \exp(\phi_{s'po})) + \frac{\lambda}{3} \sum_{k=1}^{K} \sum_{m=1}^{3} (|e_s^m|^3 + |w_p^m|^3 + |e_o^m|^3)$$
(11)

where *m* denotes the embedding part index,  $\lambda$  denotes a configurable regularisation weight parameter and |x| is the absolute value of *x*. This allows the model to answer the link prediction task in both directions: (subject, predicate, ?) and (?, predicate, object). We also consider the use of predicate reciprocals in training as described in Lacroix et. al. [5], where inverses of training predicates are added to the training set and trained with their corresponding original facts as shown in the following:

$$\mathcal{L}_{spo}^{1\text{trivec}} = -\phi_{spo} + \log(\sum_{o'} \exp(\phi_{spo'})) -\phi_{spo} + \log(\sum_{s'} \exp(\phi_{o(p+N_r)s})) + \frac{\lambda}{3} \sum_{k=1}^{K} \sum_{m=1}^{3} (|e_s^m|^3 + |w_p^m|^3 + |w_{p+N_r}^m|^3 + |e_o^m|^3)$$
(12)

where predicate  $p + N_r$  is the inverse of the predicate p where the model learns and evaluates inverse facts using inverses of their original predicates. For all the multi-class configurations, the TriVec model regularises the training facts embeddings using a dropout layer [15] with weighted probability that it learns during the grid search.

Dataset	Ent. count	Rel. count	Training	Validation	Testing
WN18	41k	18	141k	5k	5k
WN18RR	41k	11	87k	3k	3k
FB15k	15k	1k	500k	50k	60k
FB15k-237	15k	237	272k	18k	20k
YAGO10	123k	37	1M	5k	5k
NELL239	48k	239	74k	3k	3k

Table 2.	Statistics	of the	benchmarking	datasets
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## 4. Experiments

In this section, we discuss the setup of our experiments where we present the evaluation protocol, the benchmarking datasets and our implementation details.

## 4.1. Data

In our experiments we use six knowledge graph benchmarking datasets:

- WN18 & WN18RR: subsets of the WordNet dataset [16] that contains lexical information of the English language [9,10].
- FB15k & FB15k-237: subsets of the Freebase dataset [17] that contains information about general human knowledge [9,18].
- YAGO10: a subset of the YAGO3 dataset [19] that contains information mostly about people and their citizenship, gender, and professions knowledge [20].
- NELL239: a subset of the NELL dataset [7,21] that we have created as a small benchmark for fast prototyping and testing for our model, which contains general knowledge about people, places, sports teams, universities, etc.

Table 2 contains statistics about our experiments' benchmarking datasets <sup>1</sup>.

#### 4.2. Implementation

We use TensorFlow framework (GPU) along with Python 3.5 to perform our experiments. All experiments were executed on a Linux machine with processor Intel(R) Core(TM) i70.4790K CPU @ 4.00GHz, 32 GB RAM, and an nVidia Titan Xp GPU. An implementation of the proposed model is available on the github repository: https://github.com/samehkamaleldin/libkge.

# 4.3. Experiments Setup

We perform our experiments in two different configurations: (1) Ranking loss based learning: the models are trained using a ranking based loss function, where our model chooses between squared error loss and logistic loss using grid search.

<sup>&</sup>lt;sup>1</sup>All the benchmarking datasets can be downloaded using the following url: https://figshare.com/s/88ea0f4b8b139a13224f

(2) Multi-class loss based learning: the models is trained using a multi-class based training functions, where our model uses the softmax negative log loss functions described in Eq. 11 and Eq. 12.

In all of our experiments we initialise our embeddings using the Glorot uniform random generator [22] and we optimise the training loss using the Adagrad optimiser, where the learning rate  $lr \in \{0.1, 0.3, 0.5\}$ , embeddings size  $K \in 50, 75, 100, 150, 200$  and batch size  $b \in \{1000, 3000, 5000, 8000\}$  except for YAGO10 where we only use  $b \in \{1000, 2000\}$ . The rest of the grid search hyper parameters are defined as follows: in the ranking loss approach, we use the negative sampling ratio  $n \in \{2, 5, 10, 25, 50\}$  and in the multi-class approach we use regularisation weight  $\lambda \in \{0.1, 0.3, 0.35, 0.01, 0.03, 0.035\}$  and dropout  $d \in \{0.0, 0.1, 0.2, 0.01, 0.02\}$ . The number of training epochs is fixed to 1000, where in the ranking loss configuration we do an early check every 50 epochs to stop training when MRR stop improving on the validation set to prevent over-fitting.

In the evaluation process, we only consider filtered MRR and Hits@10 metrics [9]. In addition, in the ranking loss configuration, TriVec model uses a softmax normalisation of the scores of objects and subjects corruptions, that a score of a corrupted object triple  $(s, p, o_i)$  is defined as:

$$\phi_{spo_i} = \frac{\exp(\phi_{spo_i})}{\sum_{o' \in E} \exp(\phi_{spo'})},$$

similarly, we apply a softmax normalisation to the scores of all possible subject entities.

## 5. Results and Discussion

In this section we discuss findings and results of our experiments shown in Table 3 and Table 4, where the experiments are divided into two configurations: models with ranking loss functions and models with multi-class based loss functions.

## 5.1. Results of The Ranking Loss Configuration

In the results of the ranking loss configuration shown in Table 3, the results show that the TriVec model achieves best results in terms of MRR and hits@10 in five out of six benchmarking datasets with a margin of up to 10% as in the YAGO10 dataset. However, on the FB15k-237 ConvKB [23] retains state-of-the-art results in terms of MRR and Hits@10. Results also show that the factorisation based models like the DistMult, ComplEx, R-GCN and TriVec models generally outperform distance based models like the TransE and ConvKB models. However, on the FB15k-237 dataset, both distance based models outperform all other factorisation based models with a margin of up to 15% in the case of the ConvKB and the TriVec model. We intend to perform further analysis on this dataset compared to other datasets to investigate why tensor factorisation models fail to provide state-of-the-art results in future works.

Table 3. Link prediction results on standard benchmarking datasets. **\*** Results taken from [3] and our own experiments.

	Model		WN18		WN18RR		FB15k		FB15k-237		YAGO10		NELL239	
		MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	
	СР	0.08	0.13	-	-	0.33	0.53	-	-	-	-	-	-	
	TransE <b>*</b>	0.52	0.94	0.20	0.47	0.52	0.76	.29	0.48	0.27	0.44	0.27	0.43	
oss	ConvKB	-	-	0.25	0.53	-	-	0.40	0.52	-	-	-	-	
lg l	DistMult *	0.82	0.94	0.43	0.49	0.65	0.82	0.24	0.42	0.34	0.54	0.31	0.48	
nki	ComplEx *	0.94	0.95	0.44	0.51	0.70	0.84	0.22	0.41	0.36	0.55	0.35	0.52	
Ra	R-GCN	0.81	0.96	-	-	0.70	0.84	0.25	0.42	-	-	-	-	
	TriVec	0.95	0.96	0.50	0.57	0.73	0.86	0.25	0.43	0.46	0.62	0.37	0.53	

**Table 4.** Link prediction results on standard benchmarking datasets.  $\dagger$  Results taken from [5] with embedding size (*K*) limited to 200.

	Model		WN18		WN18RR		FB15k		FB15k-237		YAGO10		NELL239	
		MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	MRR	H@10	
	ConvE	0.94	0.95	0.46	0.48	0.75	0.87	0.32	0.49	0.52	0.66	0.37	0.45	
	CP-N3 †	0.12	0.18	0.08	0.14	0.35	0.56	0.22	0.42	0.40	0.64	-	-	
los	ComplEx-N3 †	0.92	0.95	0.44	0.52	0.58	0.79	0.30	0.51	0.46	0.67	-	-	
ass	CP-N3-R †	0.93	0.94	0.41	0.45	0.62	0.78	0.30	0.47	0.55	0.69	-	-	
lti-c]	ComplEx-N3-R †	0.95	0.96	0.47	0.54	0.79	0.88	0.35	0.54	0.57	0.70	-	-	
Мu	TriVec- N3	0.95	0.96	0.47	0.54	0.84	0.91	0.35	0.54	0.57	0.71	0.41	0.57	
	TriVec-N3-R	0.95	0.96	0.47	0.54	0.81	0.91	0.35	0.54	0.57	0.70	0.41	0.58	

# 5.2. Results of The Multi-class Loss Configuration

Results of the multi-class based approach show that TriVec model provide state-of-theart result on all benchmarking datasets, where the ComplEx models provide equivalent results on 3 out 6 datasets. Our reported results of the ComplEx model with multi-class log-loss introduced by Lacroix et. al. [5] are slightly different from their reported results as we re-evaluated their models with restricted embeddings size to a maximum of 200. In their work they used an embedding size of 2000, which is impractical for embedding knowledge graphs in real applications. And other previous works using the TransE, DistMult, ComplEx, ConvE, and ConvKB models have limited their experiments to a maximum embedding size of 200. In our experiments, we limited our embedding size to 200 and we have re-evaluated the models of [5] using the same restriction for a fair comparison <sup>2</sup>.

# 5.3. Ranking and Multi-class Approaches

In the link prediction task, the objective of knowledge graph embedding models is to learn embeddings that rank triples according to their faculty. This is achieved by learning to rank original true triples against other negative triple instances, where the negative instances are modelled in different ways in ranking approaches and multi-class loss approaches.

<sup>&</sup>lt;sup>2</sup>We have used the code provided at: https://github.com/facebookresearch/kbc for the evaluation of the models: CP-N3, CP-N3-R, ComplEx-N3 and ComplEx-N3-R

Model	Scoring Function	Rel. Parameters	0-Time	O-Space
RESCAL [13]	$e_s^T \mathbf{W}_r e_o$	$\mathbb{R}^{K  imes K}$	$O(K^2)$	$\mathcal{O}(K^2)$
TransE [9]	$\ e_s + w_p - e_o\ _{l1/l2}$	$\mathbb{R}^{K}$	$\mathcal{O}(K)$	$\mathcal{O}(K)$
DistMult [2]	$< e_s, w_p, e_o >$	$\mathbb{R}^{K}$	$\mathcal{O}(K)$	$\mathcal{O}(K)$
ComplEx [3]	$< e_s, w_p, \overline{e}_o >$	$\mathbb{C}^{K}$	$\mathcal{O}(K)$	$\mathcal{O}(K)$
TriVec (ours)	$< e_s^1 w_p^1 e_o^3 > + < e_s^2 w_p^2 e_o^2 > + < e_s^3 w_p^3 e_o^1 >$	$\mathbb{R}^{3K}$	$\mathcal{O}(K)$	$\mathcal{O}(K)$

In learning to rank approach, models use a ranking loss *e.g.* pointwise or pairwise loss to rank a set of true and negative instances [24], where negative instances are generated by corrupting true training facts with a ratio of negative to positive instances [9]. This corruption happens by changing either the subject or object of the true triple instance. In this configuration, the ratio of negative to positive instances is traditionally learnt using a grid search, where models compromise between the accuracy achieved by increasing the ratio and the runtime required for training.

On the other hand, multi-class based models train to rank positive triples against all their possible corruptions as a multi-class problem where the range of classes is the set of all entities. For example, training on a triple (s, p, o) is achieved by learning the right classes "s" and "o" for the pairs (?, p, o) and (s, p, ?) respectively, where the set of possible class is *E* of size  $N_e$ . Despite the enhancements of the predictions accuracy achieved by such approaches [10,5], they can have scalability issues in real-world large sized knowledge graphs with large numbers of entities due to the fact that they use the full entities' vocabulary as negative instances [25].

In summary, our model provides significantly better results than other SOTA models in the ranking setting, which is scalable and thus better-suited to real-world applications. In addition to that, our model has equivalent or slightly better performance than SOTA models on the multi-class approach.

# 6. Analysis

In this section, we study the difference between the TriVec model and KGE models in terms of scalability. We first discuss the time and space complexities of the TriVec model and other KGE models. We then execute some experiments to validate these complexities, where we discuss the relation between the training runtime of the TriVec model and other factors such as the embedding size, number of negative samples and the training data size.

# 6.1. Time and Space Complexities

Knowledge graph embedding models are known to operate with linear time and space complexity [3] as they use linear operations on vector to model embeddings interactions. However, some KGE models such as the RESCAL model [13] operate using higher complexities (quadratic time and space complexity) due to modelling embedding using matrices instead of vectors.

In our case, the TriVec model operate similar to the Complex model with an extra embedding vector. Since the ComplEx model is known to have a linear time and space

complexity [3], our model also operates with linear time and space complexity as shown in Table 5. These complexities, however, apply only to the ranking based objective settings of both the complex and TriVec models. On the other hand, the multi-class loss version of these models operate with a quadratic space complex since it use one versus all negative sampling.

# 6.2. Scalability of Training

In the following, we describe and discuss our experiments to examine the scalability of the TriVec model compared to other KGE models in relation to three factors: embedding size, negative sampling and training data size. We ran all our experiments on the KGE models with ranking based objectives.

In our experiments, we use a portion of the drug polypharmacy side-effects dataset [26] where we use the drug-drug interactions and their outcome side-effects as a knowledge graphs. In this context, our triplets are processed in the format of (drug, side-effect, drug) where drugs are modelled as entities and their resulting side effects are modelled as predicates. We then execute multiple experiments to assess the scalability of TriVec model in relation to different factors.

We first train the TransE, DistMult, ComplEx and TriVec modes on different portions of the dataset with different sizes to asses the models scalability in relation to the change of training data size. The outcomes of our experiment is illustrated in Fig. 3 plot (A) which shows that the training runtime of all the investigated models including the TriVec model grows linearly in relation to the size of the training dataset. The results also show that the ComplEx and TriVec models generally have longer training runtime than the DistMult and TransE models which can be a result of their use of multiple vectors to represent their embeddings.

Secondly, we training the KGE models on the whole a fixed size portion of the dataset using different embedding size and we report the corresponding runtime of each run in plot (B) in Fig. 3. The results show that the training time of all the models grows linearly in relation to increase of the embedding size. They also show that both the ComplEx and TriVec models have significantly higher growth factor than the DistMult and TransE models which is also a result of their use of multiple vectors to represent their embeddings as in the previous experiment.

Finally, we ran train our models with different negative sampling ratios to examine the relation between the them and the training runtime of each model. We report the results of this experiment in plot (C) in Fig. 3. The results show that all the models grow linearly in relation to the growth of the negative sampling ration while the DistMult and TransE models have relatively shorter training runtime compared to the ComplEx and TriVec models as in the previous experiments.

# 7. Example Biological Application

In this section, we present and example real life biological application of our newly proposed method, the TriVec model compared to other knowledge graph embedding techniques. We examine our method in modelling interaction networks of chemicals and human protein and their associated biological biological effects.





Figure 3. A scalability comparison between different knowledge graph embedding models in terms of the growth with relation to embedding size, negative samples and batch size.

Chemicals can have different types of effects on their target proteins such as change of expression, abundance, secretion, etc. These different effects then play various roles in the mechanism-of-action of chemicals in living systems. Therefore, understanding the chemical–protein interactions with their respective effects is crucial to elucidating the mechanism-of-action of therapeutic chemical substances.

We thus examine the of use computational methods such as the KGE models in the prediction of chemical-protein interactions and their associated biological effects. First, We build a benchmarking dataset that consists of known chemical–protein interactions with their associated chemical effects extracted from the Comparative Toxicogenomics Database (CTD) [27,28]. We them use different KGE models to generate embeddings of chemicals, their targeted proteins and corresponding chemical effects. We then use these embeddings to predict new possible chemical–protein interactions and their associated chemical effects. Finally, we show by computational experimental evaluation on our benchmark dataset that the TriVec model outperforms all other approaches in the task of predicting chemical–protein interactions and their associated chemical effects.

## 7.1. Data

We use a drug target interaction dataset extracted from the Comparative Toxicogenomics Database (CTD) [27,28]. The CTD database contains data on chemicals, pathways, diseases, exposures, genes and phenotypes. It also contains different types of associations between these entities. In our study, we only consider the chemical gene associations where we filter out the interactions according to the related species to keep only the interactions assigned to humans.

We build a new benchmarking dataset, the CTD38E dataset, which contains associations between chemicals and their human protein targets from the CTD data. It also includes the different effect types related to these associations between the chemicals and proteins. The dataset includes a set of 38 different chemical effects which are filtered according to their coverage where we only keep effects that have 500 instances or more. We further divide the dataset into training and testing splits with 90% and 10% ratios respectively for the training and evaluation pipeline.



Figure 4. A scalability comparison between different knowledge graph embedding models in terms of the growth with relation to embedding size, negative samples and batch size.

## 7.2. Experiments

In our experiments, we form the chemical-protein interactions and their effects as (chemical,effect,protein) triplets where chemicals and proteins are modelled as entities and the associated biological effects are modelled as predicates between these entities. We then train differnt standard KGE models such as the TransE, DistMult and ComplEx models along with our proposed TriVec model with its two configurations: multi-class (MC) and pointwise ranking (Pt) objectives on the training split of the benchmark dataset and evaluate it on the testing split.

Our testing split only contains true instance, therefore, we generate negative instances using uniform random sampling of chemicals and protein for each biological effect with different positive to negative ratios (1:1, 1:10 and 1:50). We then use the are under the ROC and precision recall curves to evaluate all the predictions of the KGE models on the different negative sampling ratios. We execute the evaluation of all the investigated models for each biological effect, and we then compute the averages of the two evaluation metrics to denote the models' overall metrics scores.

Further information about the problem, dataset and the use of KGE models to predict chemical-protein interactions can be found in the study of Mohamed et. al. [29].

# 7.3. Results

Fig. 4 presents a comparison between the TriVec method and other approaches in terms of predictive accuracy. The results show that the TriVec model achieves significantly better scores compared to other approaches in terms of the 1-versus-all negative to positive metrics (MRR and Hits@10). The results also show that the TriVec model with the multi-class loss approach achieves the best results in terms of all the predictive accuracy metrics.

The multi-class version of the TriVec model also achieves the best results in terms of the area under the ROC and precision recall curves. However, the difference between its scores and the scores of other models is small (ranges from 1% to 2%) in all the negative sampling configurations. The results also show that the achieved enhancements of the TriVec model positively correlate with the negative to positive ratio. This shows that the

TriVec model is able to provide better results than other approaches in harder evaluation settings (N50).

## 8. Conclusions and Future Work

In this work, we have presented the TriVec model, a new tensor factorisation based knowledge graph embedding model that represents knowledge entities an relation using three parts embeddings, where its embedding interaction function can model both symmetric and asymmetric predicates. We have shown by experiments that the TriVec model outperforms other tensor factorisation based models like the ComplEx and the DistMult on different training objectives and across all standard benchmarking datasets. We have also introduced a new challenging small size benchmarking datasets, NELL239, that can be used to facilitate fast development of new knowledge graph embedding models. We also analysed the scalability of our proposed model compared to other KGE models in terms of time and space complexity, and the scalability of its training in relation to different factors where we showed that our model have is highly scalable in training due to its linear time and space complexity.

Finally, we have compared our method to other KGE model on a real biological use case where we used the task of predicting the effects of chemical-protein interactions as a comparison application. We first built a benchmarking dataset based on the CTD database, then we used an experimental evaluation to show that our model outperforms other KGE models in predicting the biological effects of chemical-protein interactions in terms of both the area under the ROC and precision recall curves.

In our future works, we intend to investigate new possible approaches to model embedding interactions of tensor factorisation models, and we intend to analyse the effects of properties of knowledge graph datasets like FB15k-237 on the efficiency of tensor factorisation based models.

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